Holographic description of glueball and baryon in noncommutative dipole gauge theory

Wung-Hong Huang

Department of Physics, National Cheng Kung University,
No.1, University Road, Tainan, Taiwan
E-mail: `whhwung@mail.ncku.edu.tw`

ABSTRACT: We study the glueball spectrum in the supersymmetric and non-supersymmetric 4D non-commutative dipole gauge theory from the holographic description. We adopt the semiclassical WKB approximation to solve the dilaton and antisymmetric tensor field equations on the dual supergravity backgrounds to find the analytic formula of the spectrum of 0^{++} and 1^{--} glueballs, respectively. In the supersymmetric theory we see that the dipole length plays the intrinsic scale which reflects the discrete spectrum therein. In the non-supersymmetric theory, the temperature (or the radius of compactification) in there will now play the intrinsic scale and we see that the dipole has an effect to produce attractive force between the gluons within the glueball. We also study the confining force between the quarks within the baryon via strings that hang into the dipole deformed AdS geometry and see that the dipole could also produce an attractive force between the quarks. In particular, we find that the baryon has two phases in which a big baryon is dual to the static string while a small baryon is described by a moving dual string.

KEYWORDS: AdS-CFT Correspondence, Gauge-gravity correspondence.

Contents

1. Introduction

The holographics of AdS/CFT correspondence [1, 2] provides a powerful method to investigate the strong coupling gauge theory in dual supergravitional description. The correspondence has been applied to investigate several problems in large N_c QCD such as the Wilson loop [3, 4], the meson spectra/dynamics [5, 6], baryon dynamics [7–9], glueball spectrum [2, 10–14] and so on.

In the original proposal [1] the 10D background is the $AdS_5 \times O^5$. It relates to the comformally supersymmetric gauge theory which does not exist any mass scale to describe the hadronic physics and does not show confinement. Witten [2] was the first to suggest a reliable background which breaks both of the conformality and supersymmetry to describe the real physical world. In his description the AdS space is replaced by the Schwarzschild geometry describing a black hole in the AdS space. It was found that the Witten's supergravity background gives results that are in qualitative agreement with expectations for QCD at strong coupling.

In the first path of this paper we will study the spectrum of 0^{++} and 1^{--} glueball in the 4D supersymmetric and non-supersymmetric non-commutative gauge field theory

from the holographic description. The dual supergravity backgrounds are the near-horizon geometry of extremal D3-branes and nonextremal D4-branes, after applying T-duality and smeared twist, which show that a nonzero B field shall be with one leg along the brane worldvolume and other transverse to it. The solutions are the dipole deformed $AdS_5 \times O^5$ and dipole deformed Witten's supergravity background (AdS Schwarzschild spacetime) respectively, which had been constructed in $[15-17]$.

According to the holographics there is the correspondence between the chiral operators and the supergravity states [1]. For example, the operator trF^2 in four dimensions corresponds to the dilaton field of supergravity in ten dimensions. Therefore the scalar glueball $J^{\mathrm{PC}} = 0^{++}$ in QCD which couples to trF^2 is related to the dilaton propagating in the supergravity background and its mass is computable by solving the dilaton wave equation [2, 10]. In a similar way, the operator $d^{abc}F_{\mu\alpha}^a F^{b\alpha\beta} F_{\beta\nu}^c$, where d^{abc} is the symmetric structure constant, will couple to the antisymmetric tensor field and the 1^{--} glueball spectrum could be found by solving the antisymmetric tensor field wave equation [10].

In our analysis the semiclassical WKB approximation is adopted to solve the dilaton and antisymmetric tensor field equations on the supergravity background. The analytic formula of the spectrum of 0^{++} and 1^{--} glueball are therefore obtained. We see that, in the supersymmetric theory the dipole length will play the intrinsic scale which reflects the discrete spectrum in the bound states of glueball. In the non-supersymmetric theory, however, the temperature (or the radius of compactification) in there will now play the intrinsic scale and we therefore attempt to see how the dipole will modify the discrete glueball spectrum. Our analysis show that the dipole has an effect to produce attractive force between the gluons within the glueball.

In the second path of this paper we will study baryon energy. We adopt the method in [8] to study the confining force between the quarks within the baryon via strings that "hang" into the dipole deformed AdS geometry. In the method of [8] the baron vertex was considered as the strings and fivebrane which are described in terms of separated actions. The method was improved by Callen et al. [9] in which the baryon is constructed from the vertex that is considered as the D5 brane wrapped on an S^5 on which N fundamental strings terminate and they are dissolved in it [7]. The method allows an unified description of fivebrane and strings and provides an explicit string theory representation of the baryon vertex. The mathematics in Callen method is more involved and we will in this paper adopt the method of [8] for simplicity.

After the evaluation we have also seen that the dipole could produce an attractive force between the quarks within the baryon. In particular, we find that baryon has a minimum radius when the dual string is a static configuration and it will transit into another phase which dual to the moving string configuration at short distant.

In section II we follow [10] to analyze the glueball spectrum in 4D supersymmetric and non-supersymmetric dipole gauge theory. In section III we follow [8] to analyze the baryon energy. The last is devoted to a conclusion.

Note that in the non-commutative dipole field theory each field Φ_a is associated with a constant dipole length ℓ_a and we define the "non-commutative dipole product" by $\Phi_a(x) * \Phi_b(x) = \Phi_a(x-\ell_b/2) \Phi_b(x+\ell_a/2)$. It is a nonlocal field theory and break Lorentz invariance.

As there is the supergravity solution which dual to the non-commutative dipole field theory the physical particle may has nonzero dipole length therefore. The phenomenal constrain on the value of dipole length has not yet been set down now. Some properties of the noncommutative dipole field theory have been studied in [15 – 18]. The noncommutative dipole field theories are interesting by themselves and it has a chance of finding a CP violating theory [18]. It is also an appropriate candidate to study the interaction of a neutral particles with finite dipole moments, like neutrinos, with gauge particles like photons. There are some experimental evidences of such interactions, which cannot be described by the commutative version of the standard model of particles [18].

2. Glueball in dipole field theory

2.1 4D supersymmetric dipole field theory

2.1.1 Supergravity background

To find the supergravity solution dual to the noncommutative dipole theory we start with the 10D (with coordinates $t, x_1, x_2, x_3, w_a, a = 1, \ldots 6$) type II supergravity solution describing N coincident near extremal D3-brane (with worldvolume coordinates t, x_1, x_2). Now, following the prescription in [15], we first apply the T-duality transformation on the x_3 axis. Then, considering the "smeared twist" as we go around the circle of new x_3 axis (with radius R_0), i.e. the "twisted" compactification will accompany a rotation between $w_1, \ldots, w_6$ by a matrix M_{ab} in the following way:$(t, x_1, x_2, x_3, w_a) \rightarrow (t, x_1, x_2, x_3 + 2\pi R_0, \sum_{b=1}^{6} M_{ab} w_b)$, in which M is an element of the Lie algebra SO(6). After the smeared twist we finally apply the T-duality on the x_3 axis.

With a proper choice of M the dual supergravity solution used to describe the supersymmetric noncommutative dipole field theory is [15, 16]

$$
ds_{10}^2 = U^2 \left(-dt^2 + dx_1^2 + dx_2^2 + \frac{dx_3^2}{1 + B^2 U^2 \sin^2 \theta_1 \sin^2 \theta_2} \right)
$$
$$
+ \frac{1}{U^2} \left(dU^2 + U^2 d\Omega_5^2 - U^4 B^2 \sin^4 \theta_1 \sin^4 \theta_2 \frac{a_3 d\theta_3 + a_4 d\theta_4 + a_5 d\theta_5}{1 + U^2 B^2 \sin^4 \theta_1 \sin^4 \theta_2} \right).
$$
$$
e^{2\phi} = \frac{1}{1 + U^2 B^2 \sin^4 \theta_1 \sin^4 \theta_2},
$$
$$
B_{3\theta_i} = -\frac{a_i \, U^2 B \sin^4 \theta_1 \sin^4 \theta_2}{1 + U^2 B^2 \sin^4 \theta_1 \sin^4 \theta_2}, \tag{2.1}
$$

in which $a_3 \equiv \cos \theta_4$, $a_4 \equiv -\sin \theta_3 \cos \theta_3 \sin \theta_4$, and $a_5 \equiv \sin^2 \theta_3 \sin^2 \theta_4$, where θ_i are the angular coordinates parameterizing the sphere S^5 transverse to the D3 brane. The value "B" in (2.1) is proportional to the dipole length "ℓ" defined in the "non-commutative dipole product" in section I.

2.1.2 Spectrum of 0^{++} glueball

Consider first the 0^{++} glueball mass. In the supergravity description we have to solve the wave equation of dilation:

$$
\partial_\mu \left(e^{-2\phi} \sqrt{g} \, g^{\mu\nu} \, \partial_\nu \Phi \right) = 0. \tag{2.2}
$$

We look for θ_i-independent solution of the form

$$\Phi = \rho(U)e^{ik\cdot x}, \qquad k_\mu = \left(\frac{M}{\sqrt{1-\beta^2}}, 0, 0, \frac{M\beta}{\sqrt{1-\beta^2}}\right). \qquad (2.3)$$

The momentum k_μ is given by the Lorentz boost of the rest frame momentum $k_\mu = (M, 0, 0, 0)$. In other words, we consider the dilaton equation in the moving frame with the velocity β in unit of the light velocity, as that in the Moyal noncommutative theory [13]. Note that the dual string in a background with NS-NS B field is somewhat analogous to the situation when a charged particle enters a region with a magnetic field. Thus, the string will be moving with a velocity. The necessary to consider the moving dual string was first found by Maldacena in investigating the Wilson loop in Moyal-type noncommutative theory [19]. Later, it is known that the similar property also shows in the noncommutative dipole theory [16, 17].

The equation for ρ becomes

$$\partial_U \left(U^5\sqrt{1+B^2U^2}\,\partial_U\rho\right) + \frac{M^2U\sqrt{1+B^2U^2}}{\sqrt{1-\beta^2}}\left(1-\beta^2\left(1+B^2U^2\right)\right)\rho = 0. \qquad (2.4)$$

To proceed, we let $U = \sqrt{y}$ and above equation becomes

$$\partial_y \left(y^3\sqrt{1+B^2y}\,\partial_y\rho\right) + \frac{M^2\sqrt{1+B^2y}}{4\sqrt{1-\beta^2}}\left(1-\beta^2\left(1+B^2y\right)\right)\rho = 0. \qquad (2.5)$$

Next, we define $By = e^z$ and above equation becomes

$$\partial_z \left(e^{2z}\sqrt{1+e^z}\,\partial_z\rho\right) + \frac{M^2B^2e^z}{4}\sqrt{1+e^z}\,\left(1-\gamma e^z\right)\rho = 0, \qquad (2.6)$$

in which $\gamma = \beta^2/\sqrt{1-\beta^2}$. As we will solve above differential equation by the semiclassical WKB approximation we first define the wavefunction

$$\Psi(z) = \sqrt{f(z)}\,\rho(z), \qquad with \qquad f(z) = e^{2z}\sqrt{1+e^z}, \qquad (2.7)$$

then the wavefunction $\Psi(z)$ will satisfy the equation

$$\Psi''(z) + V(z)\Psi(z) = 0, \quad V(z) = \frac{M^2B^2\left(e^{-z}-\gamma\right)}{4} - \frac{1}{2}\frac{f''(z)}{f(z)} + \frac{1}{4}\left(\frac{f'(z)}{f(z)}\right)^2. \qquad (2.8)$$

In the WKB approximation we know that

$$\left(n+\frac{1}{2}\right)\pi = \int_{-\infty}^{z_0} dz\,\sqrt{V(z)}, \qquad (2.9)$$

in which z_0 is a turning point determined as following. Consider the case with $M \gg 1$ we have the approximation

$$V \approx \frac{M^2B^2}{4}\left(\left(1+\frac{3}{2M^2B^2}\right)e^{-z} - \left(\gamma + \frac{25}{4M^2B^2}\right)\right). \qquad (2.10)$$

The two turning points are therefore at $z = -\infty$ and z_0 where

$$z_0 \approx \ell n \left(\frac{1 + \frac{3}{2M^2 B^2}}{\gamma + \frac{25}{4M^2 B^2}} \right). \tag{2.11}$$

The mass spectrum evaluated from the WKB approximation in the case of $M \gg 1$ after performing the integration in (2.9) becomes

$$\left(n + \frac{1}{2} \right) \pi \approx \frac{MB\sqrt{\gamma}}{2} \left(1 + \frac{25}{8} \frac{1}{M^2 B^2 \gamma} \right), \tag{2.12}$$

which implies the following spectrum of the 0^{++} glueball:

$$M = \frac{1}{2B\sqrt{\gamma}} \left[(2n+1)\pi + \sqrt{(2n+1)^2 \pi^2 - \frac{25}{2}} \right]. \tag{2.13}$$

The dipole length B and velocity factor γ in there now play the intrinsic scale which reflects the discrete spectrum. This property is like that in [13] in which Nakajima et. al had seen that the Moyal noncommutativity could introduce an intrinsic scale in glueball discrete spectrum.

2.1.3 Spectrum of 1^{--} glueball

Consider next the 1^{--} glueball mass. In the supergravity description we have to solve the wave equation of antisymmetric tensor field $A_{\mu\nu}$:

$$\frac{3}{\sqrt{g}} \partial_{\mu_1} \left(\sqrt{g} \, \partial_{[\mu_2} A_{\mu_3 \mu_4]} \, g^{\mu_1 \mu_2} \, g^{\mu_3 \mu} \, g^{\mu_4 \nu} \right) - 16 g^{\mu_3 \mu} \, g^{\mu_4 \nu} A_{\mu_3 \mu_4} = 0. \tag{2.14}$$

As before we look for θ_i-independent solution of the form

$$A_{\mu\nu} = \rho_{\mu\nu}(U) e^{ik \cdot x}, \qquad k_\mu = \left(\frac{M}{\sqrt{1 - \beta^2}}, 0, 0, \frac{M\beta}{\sqrt{1 - \beta^2}} \right). \tag{2.15}$$

In searching the 1^{--} glueball spectrum we let $\rho_{\mu\nu}(U) = \rho(U) \, \delta_{\mu 1} \delta_{\nu 2}$ [10] and the equation for ρ becomes

$$\partial_U \left(\frac{U}{\sqrt{1 + B^2 U^2}} \, \partial_U \rho \right) + \frac{1}{U\sqrt{1 + B^2 U^2}} \left(\left(\frac{M^2}{U^2} \left(1 - \gamma B^2 U^2 \right) \right) - \frac{16}{3} \right) \rho = 0. \tag{2.16}$$

To proceed, we first let $U = \sqrt{y}$ and then define $By = e^z$, the above equation becomes

$$\partial_z \left(\frac{1}{\sqrt{1 + e^z}} \, \partial_z \rho \right) + \frac{M^2 B^2 e^{-z}}{4} \frac{1}{\sqrt{1 + e^z}} \, (1 - \gamma \, e^z) \rho = 0. \tag{2.17}$$

In the semiclassical WKB approximation the wavefunction $\Psi(z)$ defined by

$$\Psi(z) = \sqrt{f(z)} \, \rho(z), \qquad with \qquad f(z) = \frac{1}{\sqrt{1 + e^z}}, \tag{2.18}$$

will satisfy the equation

$$\Psi''(z) + V(z)\Psi(z) = 0, \quad V(z) = \frac{M^2 B^2 \left(e^{-z} - \gamma\right)}{4} - \frac{5}{16}\left(\frac{e^z}{1+e^z}\right)^2 + \frac{e^z}{4(1+e^z)} - \frac{4}{3}. \quad (2.19)$$

In the case with $M \gg 1$ we have the approximation

$$V \approx \frac{M^2 B^2}{4}\left(\left(1 + \frac{3}{2M^2 B^2}\right)e^{-z} - \left(\gamma + \frac{67}{12M^2 B^2}\right)\right). \quad (2.20)$$

The two turning points are therefore at $z = -\infty$ and z_0 where

$$z_0 \approx \ell n\left(\frac{1 + \frac{3}{2M^2 B^2}}{\gamma + \frac{67}{12M^2 B^2}}\right). \quad (2.21)$$

As before, the mass spectrum evaluated from the WKB approximation in the case of $M \gg 1$ becomes

$$\left(n + \frac{1}{2}\right) \approx \frac{MB\sqrt{\gamma}}{2}\left(1 + \frac{67}{6}\frac{1}{M^2 B^2 \gamma}\right), \quad (2.22)$$

which implies the following spectrum of the 1^{--} glueball:

$$M = \frac{1}{2B\sqrt{\gamma}}\left[(2n+1)\pi + \sqrt{(2n+1)^2\pi^2 - \frac{134}{3}}\right]. \quad (2.23)$$

The dipole length B and velocity factor γ in there also play the intrinsic scale which reflects the discrete spectrum.

As the dipole length in the 4D supersymmetric dipole theory plays the intrinsic scale we could not see how it will affect the glueball spectrum we will in next subsection investigate the 4D non-supersymmetric dipole Theory.

2.2 4D non-supersymmetric dipole field theory

2.2.1 Supergravity background

To consider the non-supersymmetric 4D dipole theory at zero temperature we shall consider the supergravity background which is constructed form near-extremal D4-brane solutions, instead of D3-brane. Follow the prescription of 2.1 we can find the proper background which is described by [17]

$$ds_{10}^2 = U^{3/2}\left[-\left(1 - \frac{U_T^3}{U^3}\right)dt^2 + dw^2 + dx^2 + dy^2 + \frac{dz^2}{1 + B^2 U^2 \sin^2\theta_1}\right] \quad (2.24)$$

$$+ \frac{1}{U^{3/2}}\left[\left(1 - \frac{U_T^3}{U^3}\right)^{-1} dU^2 + U^2 d\Omega_4^2 - B^2 U^4 \sin^4\theta_1 \frac{(a_2 d\theta_2 + a_3 d\theta_3 + a_4 d\theta_4)^2}{1 + B^2 U^2 \sin^2\theta_1}\right]$$

$$e^{2\Phi} = \frac{U^{3/2}}{1 + B^2 U^2 \sin^2\theta_1}, \quad (2.25)$$

$$B_{z\theta_i} = -\frac{a_i\, U^2 B \sin^4\theta_1}{1 + B^2 U^2 \sin^2\theta_1},$$

in which $a_2 \equiv \cos\theta_3$, $a_3 \equiv -\sin\theta_2\cos\theta_2\sin\theta_3$, and $a_4 \equiv \sin^2\theta_2\sin^2\theta_3$, where θ_i are the angular coordinates parameterizing the sphere S^4 transverse to the D4 brane. The temperature is related to U_T by

$$T = \frac{3}{4\pi}\sqrt{U_T}. \tag{2.26}$$

The value "B" in (2.24) is proportional to the dipole length "ℓ" defined in the "non-commutative dipole product" in section I.

2.2.2 Spectrum of 0^{++} glueball

To consider first the 0^{++} glueball mass in the supergravity description we have to solve the wave equation of dilation in (2.2). As before we look for θ_i-independent solution of the form

$$\Phi = \rho(U)e^{ik\cdot x}, \qquad k_\mu = (k_0, 0, 0, 0), \tag{2.27}$$

and glueball mass $M^2 = k_0^2$. The equation for ρ becomes

$$\partial_U\left(\sqrt{1+B^2U^2}\left(U^3 - U_T^3\right)U\partial_U\rho\right) + \sqrt{1+B^2U^2}\,U\,M^2\rho = 0. \tag{2.28}$$

To proceed, we let $U = \sqrt{y}$ and above equation becomes

$$\partial_y\left(\sqrt{1+B^2y}\,\left(y^{\frac{3}{2}} - y_T^{\frac{3}{2}}\right)y\,\partial_y\rho\right) + \frac{M^2}{4}\sqrt{1+B^2y}\,\rho = 0. \tag{2.29}$$

Next, we define $y = y_T(1+e^z)$ and above equation becomes

$$\partial_z\left(f(z)\,\partial_z\rho\right) + \frac{M^2e^z}{4\sqrt{y_T}}\sqrt{1+B^2y\left(1+e^z\right)}\,\rho = 0. \tag{2.30}$$

where

$$f(z) = \sqrt{1 + B^2 y_T\left(1+e^z\right)}\left(\left(1+e^z\right)^{3/2} - 1\right)\,e^{-z}\sqrt{1+e^z}. \tag{2.31}$$

In the semiclassical WKB approximation we define the wavefunction $\Psi(z) = \sqrt{f(z)}\,\rho(z)$ and the wavefunction Ψ will satisfy the equation

$$\Psi''(z) + V(z)\Psi(z) = 0, \quad V(z) = \frac{M^2}{4\sqrt{y_T}}\frac{e^{2z}}{\left(\left(1+e^z\right)^{3/2} - 1\right)\sqrt{1+e^z}} - \frac{1}{2}\frac{f''(z)}{f(z)} + \frac{1}{4}\left(\frac{f'(z)}{f(z)}\right)^2. \tag{2.32}$$

In the case of $M \gg 1$ the two turning points determined by the function $V(z)$ are at $z = -\infty$ and z_0 where

$$z_0 \approx 2\,\ell n\left[\frac{16\lambda}{9} + \left(\frac{16\lambda}{9}\right)^3\frac{B^2}{8}\right], \quad \lambda \equiv \frac{M^2}{4\sqrt{y_T}}. \tag{2.33}$$

As before, the mass spectrum evaluated from the WKB approximation becomes

$$\left(n + \frac{1}{2}\right)\pi = \int_{-\infty}^{z_0} dz\,\sqrt{\frac{\lambda e^{2z}}{\left(\left(1+e^z\right)^{3/2} - 1\right)\sqrt{1+e^z}}} = 2\sqrt{\lambda}\int_{w=1}^{\sqrt{1+e^{z_0}}}\frac{dw}{\sqrt{w^3 - 1}}$$

$$\approx 2\sqrt{\lambda}\left[\frac{2\sqrt{\pi}\Gamma\left(7/6\right)}{\Gamma\left(2/3\right)} - 2\sqrt{\frac{16\lambda}{9} + \left(\frac{16\lambda}{9}\right)^3\frac{B^2}{8}}\right], \tag{2.34}$$

which implies the following spectrum of the 0^{++} glueball:

$$M = \left(n + \frac{1}{2}\right)\pi^2 T\left[\frac{\Gamma\left(2/3\right)}{2\sqrt{\pi}\Gamma\left(7/6\right)}\right]\left(1 - \frac{B^2}{6}\frac{\Gamma\left(2/3\right)}{2\sqrt{\pi}\Gamma\left(7/6\right)}\left(\frac{\left(n+\frac{1}{2}\right)\pi}{2}\right)^{3/2}\right). \quad (2.35)$$

Therefore we see that the dipole has an effect to produce attractive force between the gluons within the glueball.

2.2.3 Spectrum of 1^{--} glueball

Consider next the 1^{--} glueball mass. In the supergravity description we have to solve the wave equation of antisymmetric tensor field $A_{\mu\nu}$ in (2.14). As before we look for θ_i-independent solution of the form

$$A_{\mu\nu} = \rho_{\mu\nu}(U)e^{ik\cdot x}, \qquad k_\mu = (k_0, 0, 0, 0). \quad (2.36)$$

In searching the 1^{--} glueball spectrum we let $\rho_{\mu\nu}(U) = \rho(U)\delta_{\mu 1}\delta_{\nu 2}$ [10] and the equation for ρ becomes

$$\partial_U\left(\frac{U^{-1/2}}{\sqrt{1+B^2U^2}}\left(U^3 - U_T^3\right)\partial_U\rho\right) + \frac{U^{-1/2}}{\sqrt{1+B^2U^2}}\left(M^2\left(1-\gamma B^2 U^2\right) - \frac{16}{3}U^{\frac{3}{2}}\right)\rho = 0. \quad (2.37)$$

As before we let $U = \sqrt{y}$, $y = y_T(1 + e^z)$ and define $\Psi(z) = \sqrt{f(z)}\,\rho(z)$, then the above equation becomes

$$\Psi''(z) + V(z)\Psi(z) = 0, \quad V(z) = V_0(z) - \frac{1}{2}\frac{f''(z)}{f(z)} + \frac{1}{4}\left(\frac{f'(z)}{f(z)}\right)^2. \quad (2.38)$$

where

$$V_0(z) = \frac{M^2}{4\sqrt{y_T}}\frac{e^{2z}}{\left((1+e^z)^{3/2}-1\right)\,(1+e^z)} - \frac{4}{3}\frac{y_T^{1/4}\,e^{2z}}{\left((1+e^z)^{3/2}-1\right)\,(1+e^z)^{1/4}} \quad (2.39)$$

$$f(z) = \frac{(1+e^z)^{1/4}\,e^{-z}}{\sqrt{1+B^2 y_T\,(1+e^z)}}\left((1+e^z)^{3/2}-1\right). \quad (2.40)$$

As before, in the case of $M \gg 1$ we can determine the two turning points and the mass spectrum of the 1^{--} glueball evaluated from the WKB approximation becomes

$$M = \left(n + \frac{1}{2}\right)\pi^2 T\left[\frac{\Gamma\left(2/3\right)}{2\sqrt{\pi}\Gamma\left(7/6\right)}\right]\left(1 - \frac{28B^2}{9}\frac{\Gamma\left(2/3\right)}{2\sqrt{\pi}\Gamma\left(7/6\right)}\left(\frac{\left(n+\frac{1}{2}\right)\pi}{2}\right)^{3/2}\right). \quad (2.41)$$

Therefore we see that the dipole has an effect to produce attractive force between the gluons within the glueball.

3. Baryon in dipole field theory

The dual baryon configuration suggested in [7, 8] contains two contributions. The first is the string stretched between the boundary of the AdS_5 space and the second is the D5-brane wrapped on the S^5. They are of the same order and we should consider both of them.

The action for a static D5-brane wrapped on dipole deformed S^5 in (2.1) is

$$S_{D_5} = \frac{1}{(2\pi)^5} \int dx^6 e^{-\phi} \sqrt{g} = \frac{TNU_0}{8\pi} + O(B^4), \tag{3.1}$$

in which we neglect the terms higher then B^2 order. Note that T is the time period and U_0 is the location of the baryon vertex in the bulk.

The N strings we considered is such that the strings end on boundary of the AdS_5 space with radius L in a symmetric way. The string may be static or moving under the NS-NS B field, which are investigated in the follow.

3.1 Static string configuration

The static string described by the Nambu-Goto action in the gauge

$$t = \tau, \qquad z = \sigma, \qquad U = U(z), \tag{3.2}$$

is

$$S_{F_1} = \frac{T}{2\pi} \int dz \sqrt{(\partial_z U)^2 + \frac{U^4}{R^4 (1 + B^2 U^2)}}. \tag{3.3}$$

The total action is the summation of (3.1) and (3.3) and the variation of it under $U \to U + \delta U$ contains a volume term as well as a surface term. The volume term leads to

$$\frac{\frac{U^4}{(1+B^2U^2)}}{\sqrt{(\partial_z U)^2 + \frac{U^4}{R^4(1+B^2U^2)}}} = constant, \tag{3.4}$$

because the Lagrangian of action S_{F_1} does not depend explicitly on z. The surface term yields the relation

$$\delta U \frac{TN}{8\pi} = \delta U \frac{TN(\partial U_0)}{2\pi \sqrt{(\partial_z U_0)^2 + \frac{U_0^4}{R^4 \left(1 + B^2 U_0^2\right)}}}, \tag{3.5}$$

which implies that

$$(\partial_z U_0)^2 = \frac{U_0^4}{15 R^4 \left(1 + B^2 U_0^2\right)}. \tag{3.6}$$

Using this relation and consider the constant in (3.4) as the value at U_0 we find that

$$(\partial_z U)^2 = \frac{U^4}{R^4 (1 + B^2 U^2)} \left[\frac{16}{15} \frac{\left(1 + B^2 U_0^2\right) U^4}{\left(1 + B^2 U^2\right) U_0^4} - 1 \right]. \tag{3.7}$$
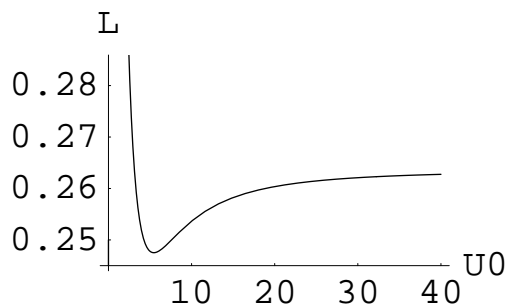
Figure 1: The function $L(U_0)$ at $B = 0.2$. We see that baryon has a minimum radius.

With the help of this relation the radius of the baryon could be calculated by

$$
\begin{aligned}
L = \int dz = \int \frac{dU}{\partial_z U} &= \int_{U_0}^{\infty} dU \; \frac{R^2 \sqrt{1 + B^2 U^2}}{U^2 \sqrt{\frac{16}{15} \frac{(1+B^2 U_0^2) U^4}{(1+B^2 U^2) U_0^4} - 1}} \\
&= \frac{R^2}{U_0} \int_1^{\infty} dy \; \frac{\sqrt{1 + B^2 U_0^2 y^2}}{y^2 \sqrt{\frac{16}{15} \frac{(1+B^2 U_0^2) y^4}{1 + B^2 U_0^2 y^2} - 1}} \\
&= \frac{R^2}{U_0} \int_0^1 dx \; \frac{x^2 + B^2 U_0^2}{\sqrt{\frac{16}{15}\left(1 + B^2 U_0^2\right) - x^2 \left(x^2 + B^2 U_0^2\right)}}.
\end{aligned}
\tag{3.8}
$$

Substituting the relation (3.7) into (3.3) the part of the baryon energy that comes from the N string could be evaluated by the formula

$$
M_{F1} = \frac{U_0}{2\pi} \int_1^{\infty} dy \; \left(\frac{\sqrt{\frac{16}{15}} \sqrt{1 + B^2 U_0^2} \; y^2}{\sqrt{\frac{16}{15}\left(1 + B^2 U_0^2\right) y^4 - \left(1 + B^2 U_0^2 y^2\right)}} - 1 \right) - \frac{U_0}{2\pi}.
\tag{3.9}
$$

We can therefore use the equations (3.8) and (3.9) to find the value of M_{F1} which becomes the function of baryon radius L.

3.1.1 Small baryon: minimum radius

A numerical evaluation of (3.8) is presented in figure 1.

Thus we see that the baryon with a dipole will has a minimum radius. The existence of the minimum radius could also be directly analyzed from (3.8).

In the limit of $U_0 \to \infty$ eq. (3.8) gives the following approximation

$$
L(U_0 \to \infty) \approx \frac{R^2}{U_0} \int_0^1 dx \; \frac{B^2 U_0^2}{B U_0 \sqrt{\frac{16}{15} - x^2}} = R^2 B \sin^{-1}\left(\sqrt{\frac{16}{15}} \right).
\tag{3.10}
$$

Thus the minimum radius is proportional to the dipole length B and $\sqrt{N}$ (note that $R^4 = 4\pi N$). However, it shall be noticed that, form figure 1 we see that the real minimum value of the baryon radius is less then above value.

3.1.2 Big baryon

The big baryon with large radius L is that with small U_0. At small value of U_0 the equations (3.8) and (3.9) could be approximated as

$$L \approx \frac{R^2}{U_0} \left[\int_0^1 dx \, \frac{x^2}{\sqrt{\frac{16}{15} - x^4}} + B^2 U_0^2 \int_0^1 dx \, \left[\frac{1}{\sqrt{\frac{16}{15} - x^4}} - \frac{1}{2} \frac{x^2(\frac{16}{15} - x^2)}{\left(\frac{16}{15} - x^4\right)^{3/2}} \right] \right]$$

$$= \frac{R^2}{U_0} \left[0.481 + 0.994 B^2 U_0^2 \right], \tag{3.11}$$

$$M_{F1} \approx \frac{U_0}{2\pi} \int_1^\infty dy \, \left[\frac{\sqrt{\frac{16}{15}} y^2}{\sqrt{\frac{16}{15} y^4 - 1}} + B^2 U_0^2 \sqrt{\frac{16}{15}} \frac{y^2}{2} \left(\frac{1}{\sqrt{\frac{16}{15} y^4 - 1}} - \frac{\frac{16}{15} y^4 - y^2}{\left(\frac{16}{15} y^4 - 1\right)^{3/2}} \right) - 1 \right] - \frac{U_0}{2\pi}$$

$$= \frac{U_0}{2\pi} \left[0.284 + 0.398 B^2 U_0^2 \right] - \frac{U_0}{2\pi}. \tag{3.12}$$

We can use (3.11) to express U_0 as function L

$$U_0 = \frac{R^2}{L} \left[0.481 + 0.230 B^2 \frac{R^4}{L^2} \right]. \tag{3.13}$$

Substituting above relation into (3.12) and (3.1) we finally find the baryon energy

$$H \equiv M_{F1} + M_{D_5} \approx \frac{1}{2\pi} \left(-\frac{0.224 R^2}{L} - \frac{0.063 R^6}{L^3} B^2 \right), \tag{3.14}$$

in which M_{D_5} is the energy contribution from D5 brane in (3.1). Above results tell us that dipole could produce an attractive force between the quarks to reduce the baryon energy.

As mentioned in section II, the string in a background with B field is somewhat analogous to the situation when a charged particle enters a region with a magnetic field. Thus, the string may be moving with a velocity. So let us consider the case with moving string in below.

3.2 Moving string configuration

The moving string we considered is described by the gauge [20]

$$t = \tau, \quad \theta_3 = \omega t, \quad z = \sigma, \quad U = U(z). \tag{3.15}$$

The Nambu-Goto action in this gauge is

$$S_{F_1} = \frac{T}{2\pi} \int dz \, \sqrt{\left(1 - \frac{R^4}{U^2} \frac{\omega^2}{1 + B^2 U^2} \right) \left((\partial_z U)^2 + \frac{U^4}{R^4 (1 + B^2 U^2)} \right)} - \frac{B \omega U^2}{1 + B^2 U^2}. \tag{3.16}$$

As before, the variation of volume term gives

$$\frac{U^4}{R^4 (1 + B^2 U^2)} \frac{\sqrt{1 - \frac{R^4}{U^2} \frac{\omega^2}{1 + B^2 U^2}}}{\sqrt{(\partial_z U)^2 + \frac{U^4}{R^4 (1 + B^2 U^2)}}} + \frac{B \omega U^2}{1 + B^2 U^2} = constant, \tag{3.17}$$

because the Lagrangian of action S_{F_1} does not depend explicitly on z. The variation of surface term gives

$$(\partial_z U_0)^2 = \frac{U_0^4}{R^4 \left(15 \left(1 + B^2 U_0^2\right) - \frac{16R^4 \omega^2}{U_0^2}\right)}. \tag{3.18}$$

Using this relation and consider the constant in (3.4) as the value at U_0 we have a relation

$$(\partial_z U)^2 = \frac{\frac{U^8}{R^8(1+B^2U^2)^2} \left(1 - \frac{R^4}{U^2} \frac{\omega^2}{1+B^2U^2}\right)}{\left(\frac{\sqrt{15\left(1+B^2U_0^2\right)-\frac{16R^4\omega^2}{U_0^2}}}{4} \frac{U_0^2}{R^2\left(1+B^2U_0^2\right)} + \frac{B\omega U_0^2}{1+B^2U_0^2} - \frac{B\omega U^2}{1+B^2U^2}\right)^2} - \frac{U^4}{R^4 \left(1 + B^2 U^2\right)}. \tag{3.19}$$

Using above relation we will determine the angular velocity ω.

In the limit $U \to \infty$ above relation becomes

$$(\partial_z U)^2 \to \frac{\frac{U^4}{R^8 B^4}}{\left[\frac{\sqrt{15\left(1+B^2U_0^2\right)-\frac{16R^4\omega^2}{U_0^2}}}{4} \frac{U_0^2}{R^2\left(1+B^2U_0^2\right)} + \frac{B\omega U_0^2}{1+B^2U_0^2} - \frac{\omega}{B}\right]^2}. \tag{3.20}$$

Thus, if the bracket term in above equation is zero then, for the fixed values of U_0 and B there will have a minimum radius L as can be seen from the relation $L = \int dz = \int \frac{dU}{\partial_z U}$ which has been used in (3.8). While the dipole has effect to produce attractive force the baryon with small radius will therefore has less energy. Thus the least energy of dual moving string will rotate with an angular velocity which is the solution by letting the bracket term in (3.20) to be zero. This property give a very simple relation between angular velocity ω and dipole length B:

$$\omega^2 = \frac{15B^2}{16} \frac{U_0^4}{R^4} \tag{3.21}$$

Substituting this relation into (3.19) we also find a very simple relation

$$(\partial_z U)^2 = \frac{U^4}{R^4} \left[\frac{16}{15} \frac{U^4}{U_0^4} - 1\right]. \tag{3.22}$$

This relation is just (3.7) while let $B = 0$. Thus we find that a moving string has the same result as that without dipole field. This seems a surprise property at first sight. The reason behind it may be argued as following.

The dual string in a background with $B_{z\theta_3}$ field is somewhat analogous to the situation when a charged particle enters a region with a magnetic field. Thus, the string will be rotating along θ_3 with a constant angular momentum ω which is proportional to the strength of the NS-NS field, as shown in (3.21). The configuration described in (3.15) has a binding energy (it is negative) from B field which will be just canceled by the kinetic energy (it is positive) from the moving. Thus the moving dual string does not depend on the value of dipole field and we have the same result as that without dipole field.

For clear we plot in figure 2 the baryon energy calculated from static and moving strings.
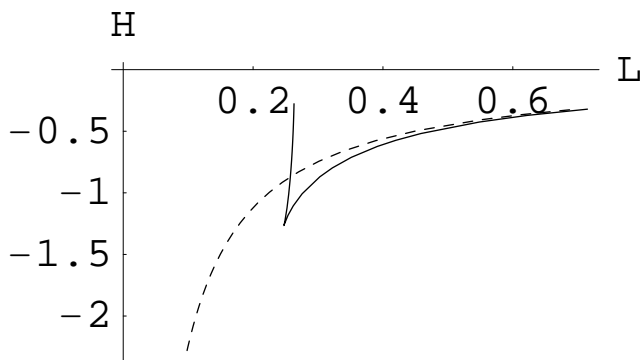
Figure 2: The baryon energy calculated from static string (in solid line) and moving string (in dashed line) at $B = 0.2$. We see that a small baryon could be found in the dual moving string which then transit to the static string configuration of big baryon.

In conclusion, as the static string configuration shows an attractive force the baryon will therefore be in the static string configuration which, however could exist only if it has a radius larger then a critical value. A small baryon could be found in the dual moving string which is that transit from the static configuration as shown in figure 2. The transition is first order as the energy of the two configurations at transition radius is discontinuous.

4. Conclusion

In this paper, we first follow [10] to investigate the glueball spectrum in the supersymmetric and non-supersymmetric 4D non-commutative dipole gauge theory from the holographic description. To find the analytic formula of the spectrum of 0^{++} and 1^{--} glueball we adopt the semiclassical WKB approximation to solve the dilaton and antisymmetric tensor field equations on the dual supergravity backgrounds, respectively. In the supersymmetric theory we see that the dipole length plays the intrinsic scale which reflects the discrete spectrum therein. In the non-supersymmetric theory, the temperature (or the radius of compactification) in there will now play the intrinsic scale and we see that the dipole has an effect to produce attractive force between the gluons within the glueball.

We next study the baryon energy following the method in [8]. To study the confining force between the quarks within the baryon we consider the strings that "hang" into the dipole deformed AdS geometry and regarded the baron vertex as the strings and fivebrane which are described in terms of separated actions. We first consider the string as a static configuration and find that the baryon could only exist if it is larger then a critical radius. We see that the dipole could also produce an attractive force between the quarks. We next consider the string as a configuration moving with an angular velocity. We find that the angular velocity is proportional to the NS-NS B field. After the evaluation we find that the baryon has two phases in which a big baryon is dual to the static string while a small baryon is described by a moving dual string. The phase transition property is like

that in our study of the Wilson loop of non-commutative gauge theory form dual string description [20].

Finally, the baryon vertex constructed by Callen et al. [9] is considered as the D5 brane wrapped on an S^5 on which N fundamental strings terminate and they are dissolved in it [7]. While the method could be used to investigate the more details of the baryon vertex the mathematics therein is quite involved. It is interesting to see how the property of phase transition from small baryon to big baryon would be shown in the Callen method. We will investigate this problem in the next paper. The properties of glueball and baryon on the other dipole field deformed background [21] are of interesting and remain to be studied.

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [`hep-th/9711200`];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [`hep-th/9802109`];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [`hep-th/9802150`].

[2] E. Witten, *Anti-de Sitter space, thermal phase transition and confinement in gauge theories*, *Adv. Theor. Math. Phys.* **2** (1998) 505 [`hep-th/9803131`].

[3] J.M. Maldacena, *Wilson loops in large-N field theories*, *Phys. Rev. Lett.* **80** (1998) 4859 [`hep-th/9803002`];
S.-J. Rey and J.-T. Yee, *Macroscopic strings as heavy quarks in large-N gauge theory and Anti-de Sitter supergravity*, *Eur. Phys. J.* **C 22** (2001) 379 [`hep-th/9803001`].

[4] N. Drukker and B. Fiol, *All-genus calculation of Wilson loops using D-branes*, *JHEP* **02** (2005) 010 [`hep-th/0501109`];
S.A. Hartnoll and S. Prem Kumar, *Multiply wound Polyakov loops at strong coupling*, *Phys. Rev.* **D 74** (2006) 026001 [`hep-th/0603190`];
S. Yamaguchi, *Wilson loops of anti-symmetric representation and D5-branes*, *JHEP* **05** (2006) 037 [`hep-th/0603208`].

[5] A. Karch and E. Katz, *Adding flavor to AdS/CFT*, *JHEP* **06** (2002) 043 [`hep-th/0205236`];
A. Karch, E. Katz and N. Weiner, *Hadron masses and screening from AdS Wilson loops*, *Phys. Rev. Lett.* **90** (2003) 091601 [`hep-th/0211107`];
M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, *Meson spectroscopy in AdS/CFT with flavour*, *JHEP* **07** (2003) 049 [`hep-th/0304032`];
J. Babington, J. Erdmenger, N.J. Evans, Z. Guralnik and I. Kirsch, *Chiral symmetry breaking and pions in non-supersymmetric gauge/gravity duals*, *Phys. Rev.* **D 69** (2004) 066007 [`hep-th/0306018`].

[6] J. Erdmenger, N. Evans, I. Kirsch and E. Threlfall, *Mesons in gauge/gravity duals — A review*, *Eur. Phys. J.* **A35** (2008) 81 [`arXiv:0711.4467`].

[7] E. Witten, *Baryons and branes in Anti de Sitter space*, *JHEP* **07** (1998) 006 [`hep-th/9805112`];
D.J. Gross and H. Ooguri, *Aspects of large-N gauge theory dynamics as seen by string theory*, *Phys. Rev.* **D 58** (1998) 106002 [`hep-th/9805129`].

[8] A. Brandhuber, N. Itzhaki, J. Sonnenschein and S. Yankielowicz, *Baryons from supergravity*, *JHEP* **07** (1998) 020 [`hep-th/9806158`];
Y. Imamura, *Baryon mass and phase transitions in large-N gauge theory*, *Prog. Theor. Phys.* **100** (1998) 1263 [`hep-th/9806162`].

[9] Y. Imamura, *Supersymmetries and BPS configurations on Anti-de Sitter space*, *Nucl. Phys.* **B 537** (1999) 184 [`hep-th/9807179`];
C.G. Callan Jr., A. Guijosa and K.G. Savvidy, *Baryons and string creation from the fivebrane worldvolume action*, *Nucl. Phys.* **B 547** (1999) 127 [`hep-th/9810092`];
C.G. Callan Jr., A. Guijosa, K.G. Savvidy and O. Tafjord, *Baryons and flux tubes in confining gauge theories from brane actions*, *Nucl. Phys.* **B 555** (1999) 183 [`hep-th/9902197`].

[10] C. Csáki, H. Ooguri, Y. Oz and J. Terning, *Glueball mass spectrum from supergravity*, *JHEP* **01** (1999) 017 [`hep-th/9806021`]; *Large-N QCD from rotating branes*, *Phys. Rev.* **D 59** (1999) 065012 [`hep-th/9810186`];
R. de Mello Koch, A. Jevicki, M. Mihailescu and J.P. Nunes, *Evaluation of glueball masses from supergravity*, *Phys. Rev.* **D 58** (1998) 105009 [`hep-th/9806125`];
J.A. Minahan, *Glueball mass spectra and other issues for supergravity duals of QCD models*, *JHEP* **01** (1999) 020 [`hep-th/9811156`].

[11] R.C. Brower, S.D. Mathur and C.-I. Tan, *Glueball spectrum for QCD from AdS supergravity duality*, *Nucl. Phys.* **B 587** (2000) 249 [`hep-th/0003115`].

[12] H. Boschi-Filho and N.R.F. Braga, *Gauge/string duality and scalar glueball mass ratios*, *JHEP* **05** (2003) 009 [`hep-th/0212207`];
H. Boschi-Filho, N.R.F. Braga and H.L. Carrion, *Glueball Regge trajectories from gauge/string duality and the Pomeron*, *Phys. Rev.* **D 73** (2006) 047901 [`hep-th/0507063`];
R. Apreda, D.E. Crooks, N.J. Evans and M. Petrini, *Confinement, glueballs and strings from deformed AdS*, *JHEP* **05** (2004) 065 [`hep-th/0308006`].

[13] T. Nakajima, K. Suzuki and H. Takahashi, *Glueball mass spectra for supergravity duals of noncommutative gauge theories*, *JHEP* **01** (2006) 016 [`hep-th/0508054`].

[14] P. Colangelo, F. De Fazio, F. Jugeau and S. Nicotri, *On the light glueball spectrum in a holographic description of QCD*, *Phys. Lett.* **B 652** (2007) 73 [`hep-ph/0703316`];
Investigating AdS/QCD duality through scalar glueball correlators, `arXiv:0711.4747`.

[15] A. Bergman and O.J. Ganor, *Dipoles, twists and noncommutative gauge theory*, *JHEP* **10** (2000) 018 [`hep-th/0008030`];
A. Bergman, K. Dasgupta, O.J. Ganor, J.L. Karczmarek and G. Rajesh, *Nonlocal field theories and their gravity duals*, *Phys. Rev.* **D 65** (2002) 066005 [`hep-th/0103090`].

[16] M. Alishahiha and H. Yavartanoo, *Supergravity description of the large-N noncommutative dipole field theories*, *JHEP* **04** (2002) 031 [`hep-th/0202131`].

[17] W.-H. Huang, *Wilson-t' Hooft loops in finite-temperature non-commutative dipole field theory from dual supergravity*, *Phys. Rev.* **D 76** (2007) 106005 [`arXiv:0706.3663`].

[18] K. Dasgupta and M.M. Sheikh-Jabbari, *Noncommutative dipole field theories*, *JHEP* **02** (2002) 002 [`hep-th/0112064`];
N. Sadooghi and M. Soroush, *Noncommutative dipole QED*, *Int. J. Mod. Phys.* **A 18** (2003) 97 [`hep-th/0206009`].

[19] J.M. Maldacena and J.G. Russo, *Large-N limit of non-commutative gauge theories*, *JHEP* **09** (1999) 025 [hep-th/9908134];
A. Hashimoto and N. Itzhaki, *Non-commutative Yang-Mills and the AdS/CFT correspondence*, *Phys. Lett.* **B 465** (1999) 142 [hep-th/9907166].

[20] W.-H. Huang, *Dual string description of Wilson loop in non-commutative gauge theory*, *Phys. Lett.* **B 647** (2007) 519 [*Erratum ibid.* **652** (2007) 388] [hep-th/0701069].

[21] U. Gürsoy and C. Núñez, *Dipole deformations of N = 1 SYM and supergravity backgrounds with* U(1) × U(1) *global symmetry*, *Nucl. Phys.* **B 725** (2005) 45 [hep-th/0505100];
N.P. Bobev, H. Dimov and R.C. Rashkov, *Semiclassical strings, dipole deformations of N = 1 SYM and decoupling of KK modes*, *JHEP* **02** (2006) 064 [hep-th/0511216].